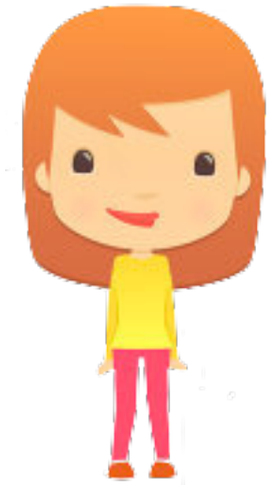
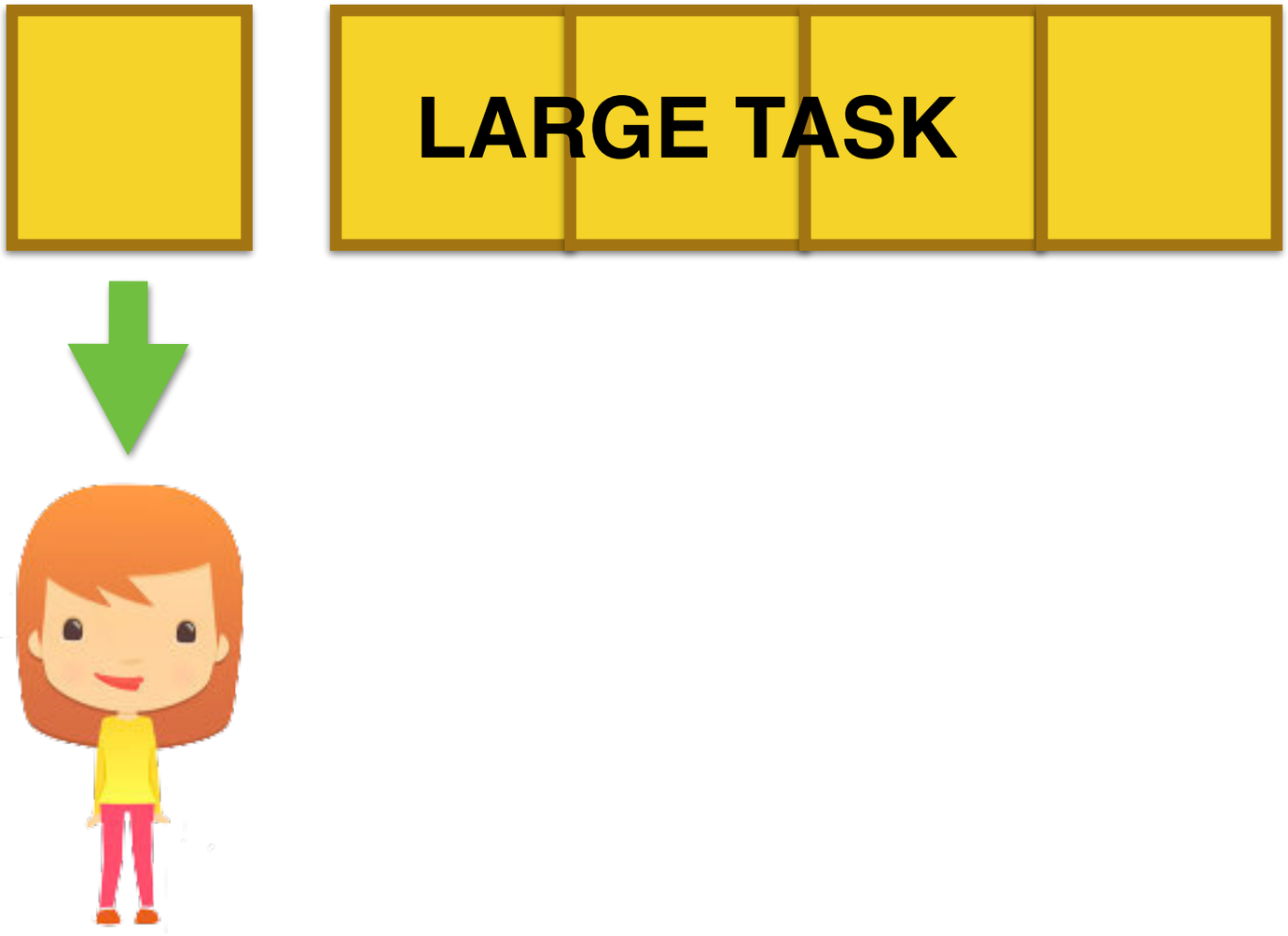
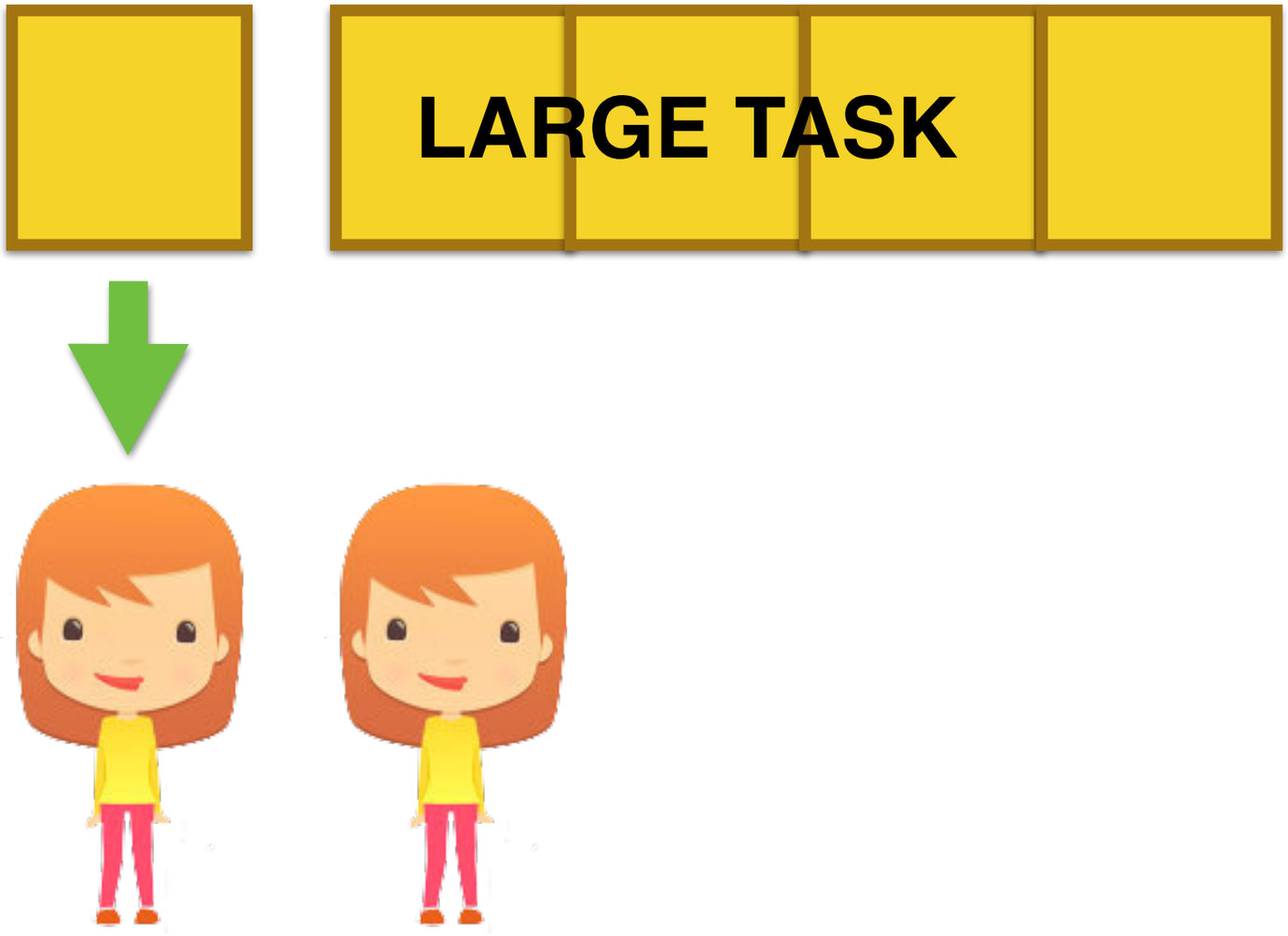
Introduction To Recursive Functions

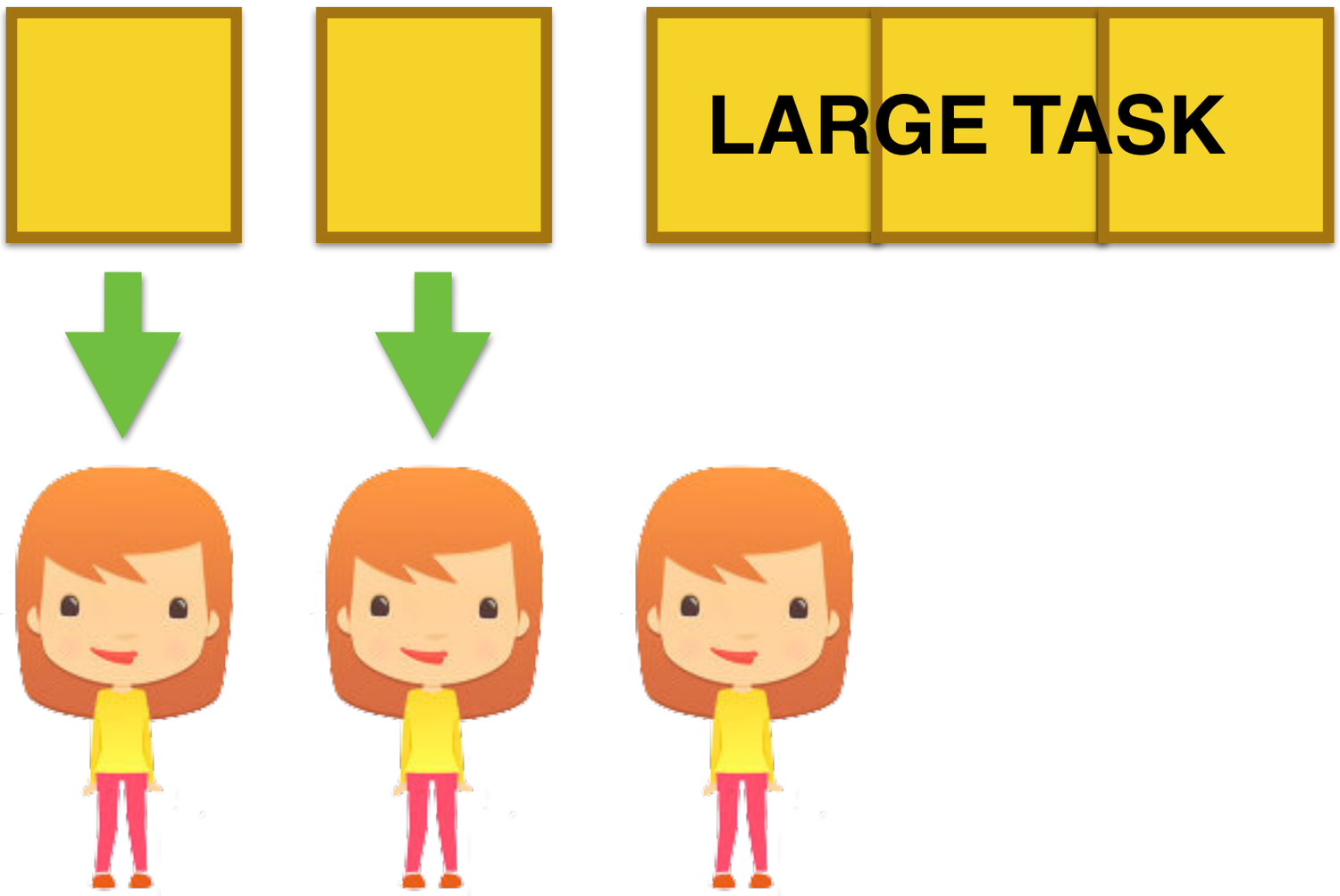
D. Thiebaut — CSC212
Fall 2014

LARGE TASK

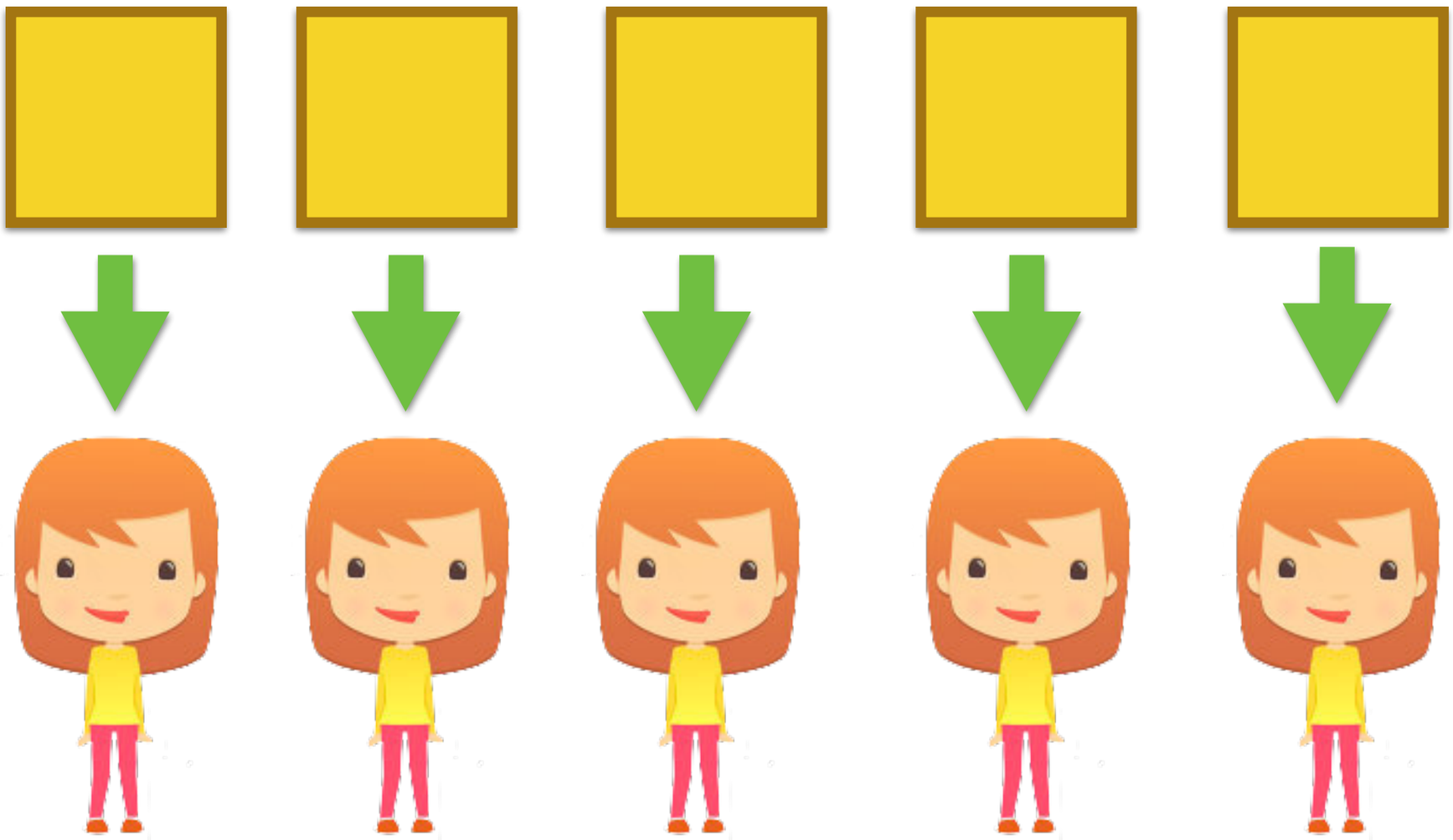


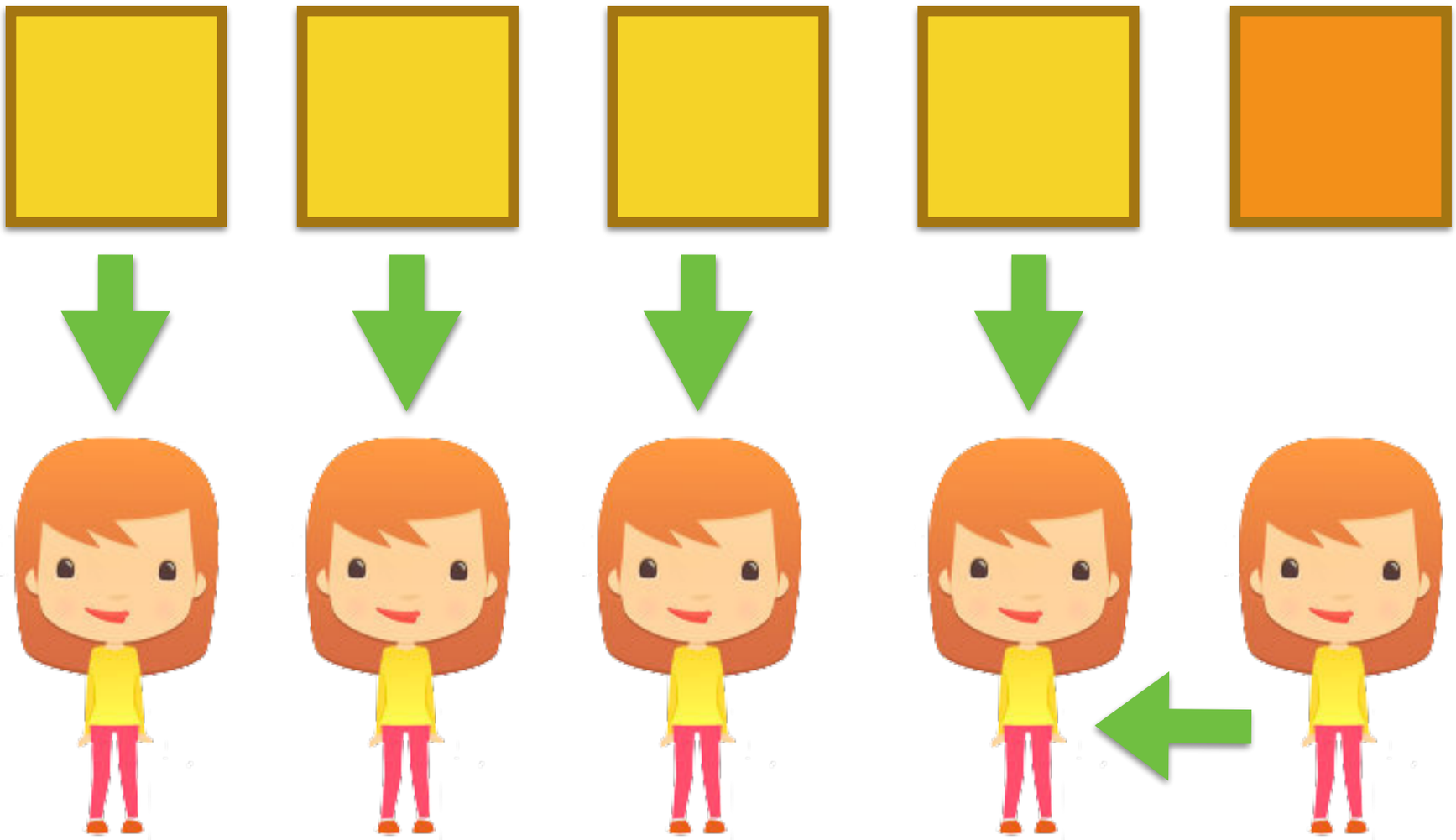

LARGE TASK

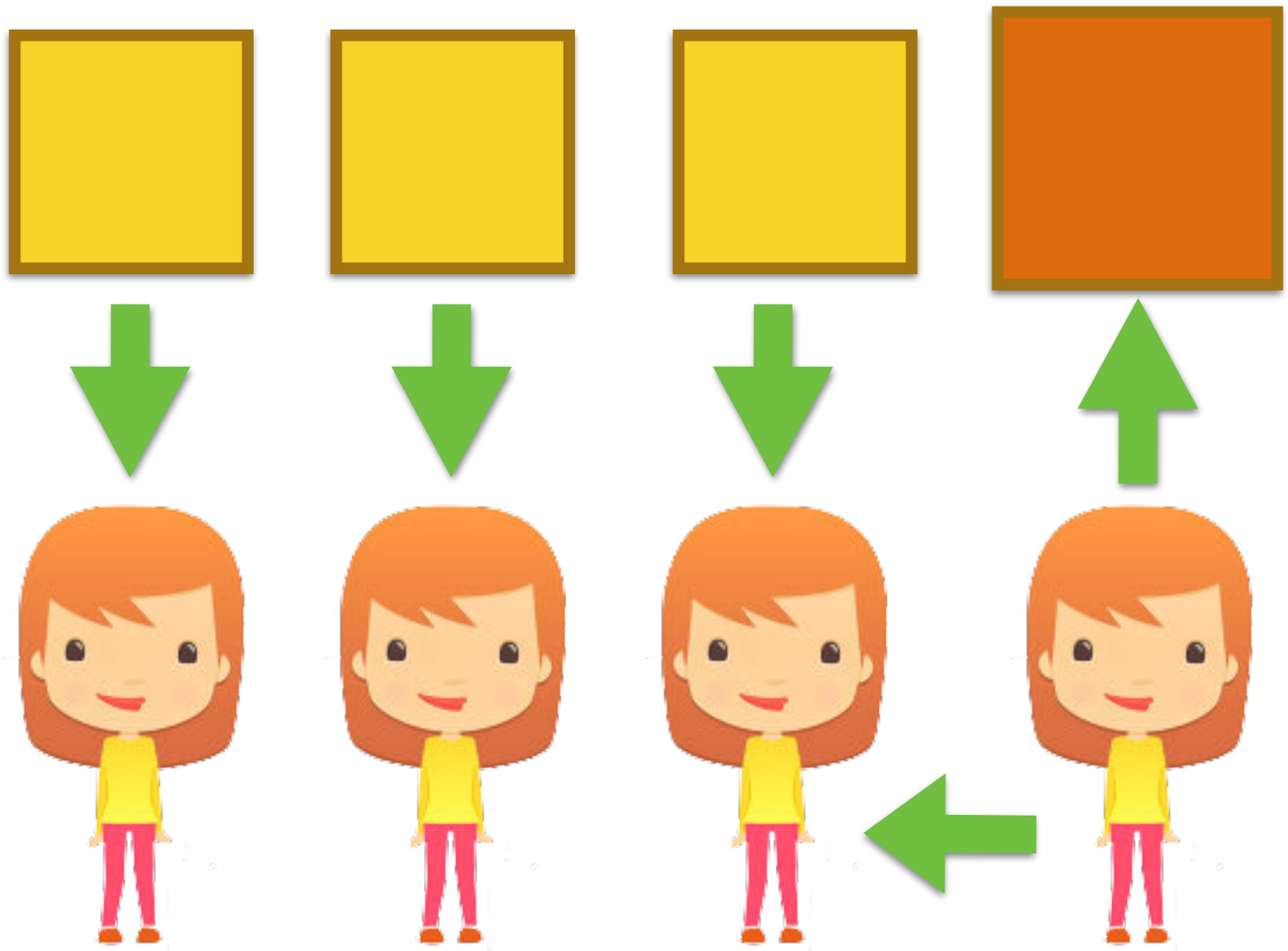

LARGE TASK

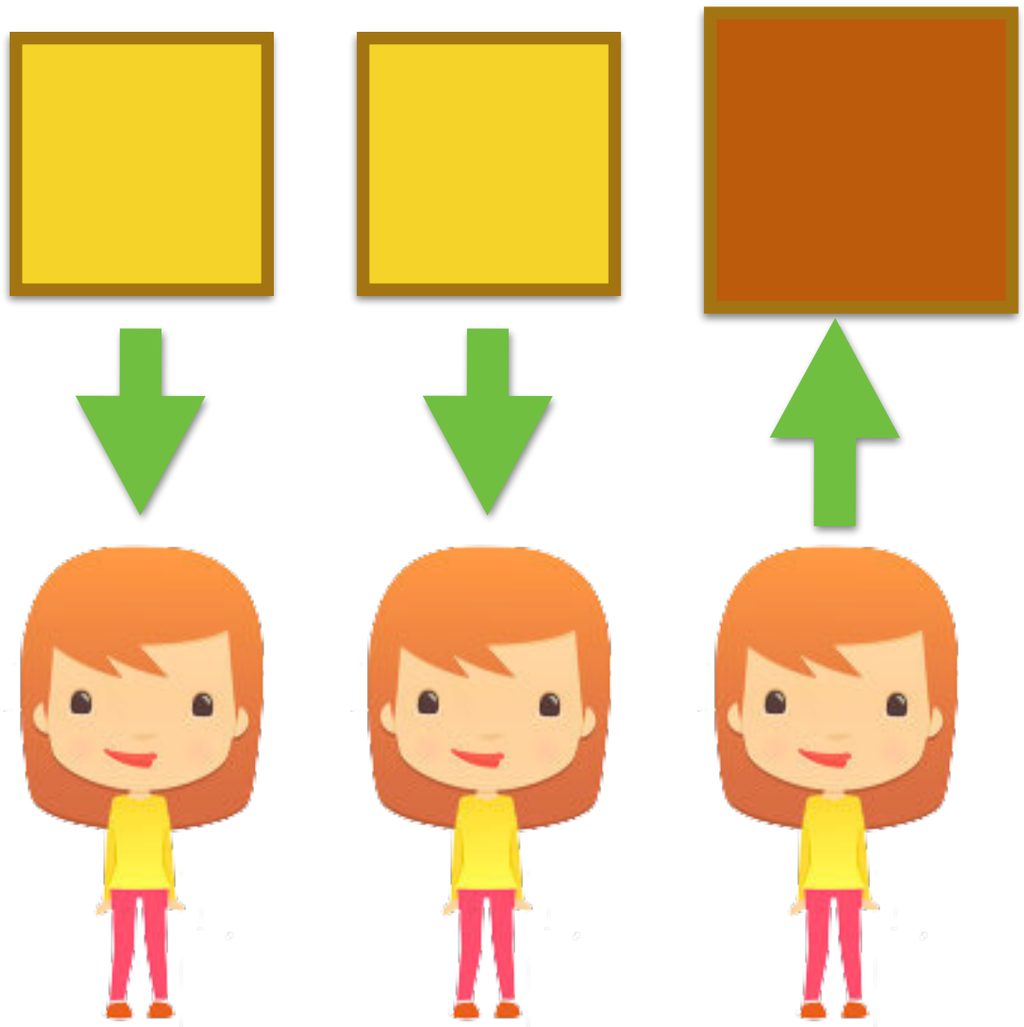

LARGE TASK

AFTER A WHILE…

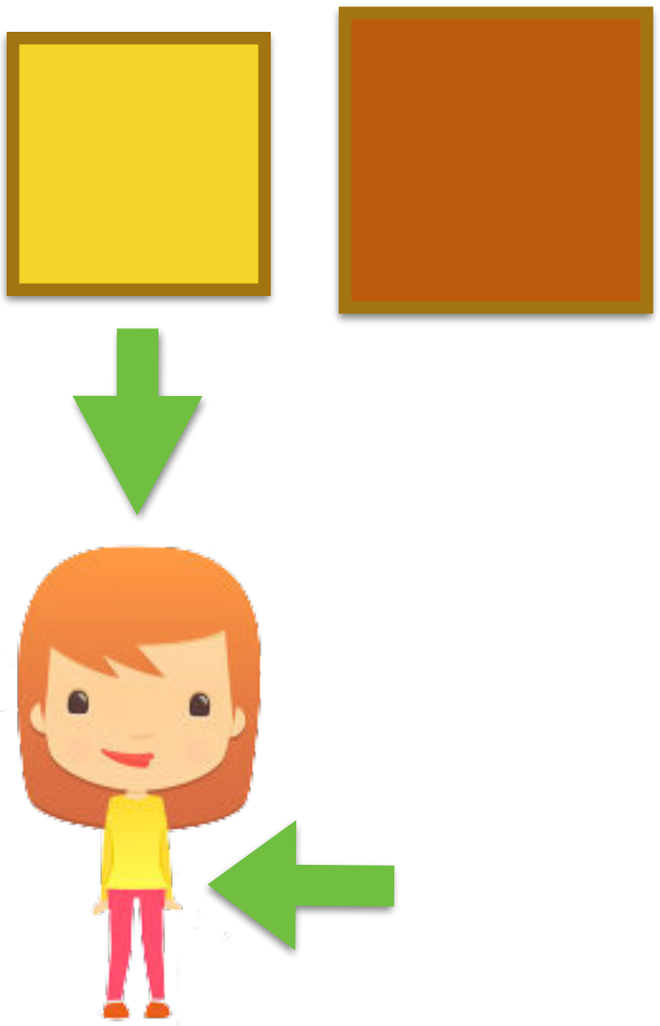








AFTER A WHILE…



THE ANSWER!

Important Concept: 1

- Recursive step reduces the problem in a small, but significant way, getting closer to a solution

- Work done during a recursive call builds up on the partial solution found so far.

Important Concept: 2

- Recursion requires

  1. **Stopping** Condition

  2. Recursive Step **Reducing Size of Problem** and leading closer to solution.

Examples

Draft Algorithm, then Code

- **Factorial**
- **Sum** up an array
  - from N go 1+(N-1)
    - from N go N/2 and N/2
- Find the **largest** element of an array
- Find a key in an **unsorted** array
- Find a key in a **sorted** array (binary search)
- Evaluate an **RPN** expression

Evaluating Time Complexity

- Factorial

```
private static int factorial(int n) {
   if ( n <= 1 )
      return 1;

   return n * factorial( n - 1 );
}
```

Evaluating Time Complexity

- Binary Search

```
private static int binSearch( ArrayList A, int low, int high, int key ) {
    if ( low > high )
        return -1;

    int mid = ( low+high )/2;
    if ( (int) A.get( mid ) == key )
        return mid;

    if ( (int) A.get( mid ) < key )
        return binSearch( A, mid+1, high, key );
    else
        return binSearch( A, low, mid-1, key );
}
```

Recursion is Not Required

```
private static int loopingBinSearch( int key, int[] A ) {
    int l = 0, h = A.length-1;
    int index = -1;

    while ( l <= h ) {
        int m = (l+h)/2;
        if ( A[m] == key ) {
            index = m;
            break;
        }
        if ( key < A[m] )
            h = m-1;
        else
            l = m+1;
    }
    return index;
}
```

Non-recursive version of BinarySearch
using a while-loop to move
the “low” and “high” indexes…

Tail Recursion

Let's Revisit Fibonacci

```
private static long computeFibRecursively( int n ) {

      if ( n <= 1 )
       return 1;

   return computeFibRecursively( n-1 ) + computeFibRecursively( n-2 );
}
```

Fib's Call Tree

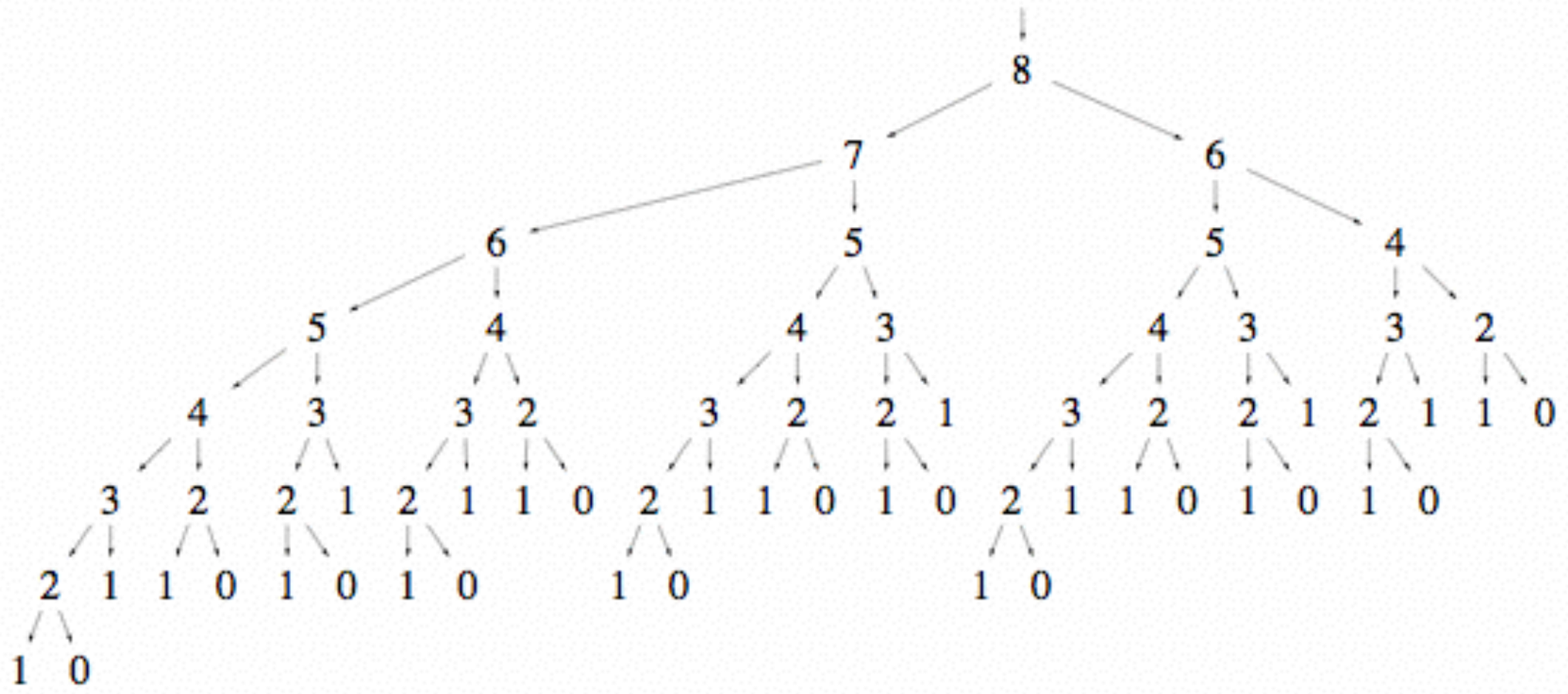
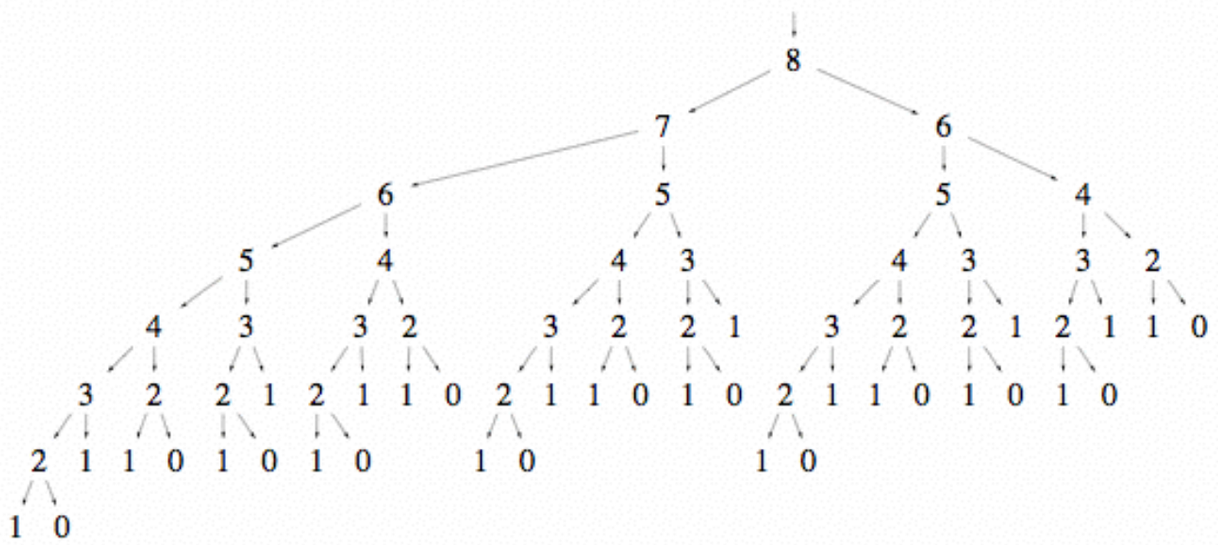


Image taken from http://www.cs.ucr.edu/~neal/2005/cs141/wikidb/uploads/fib_call_graph_8.gif

Observations:



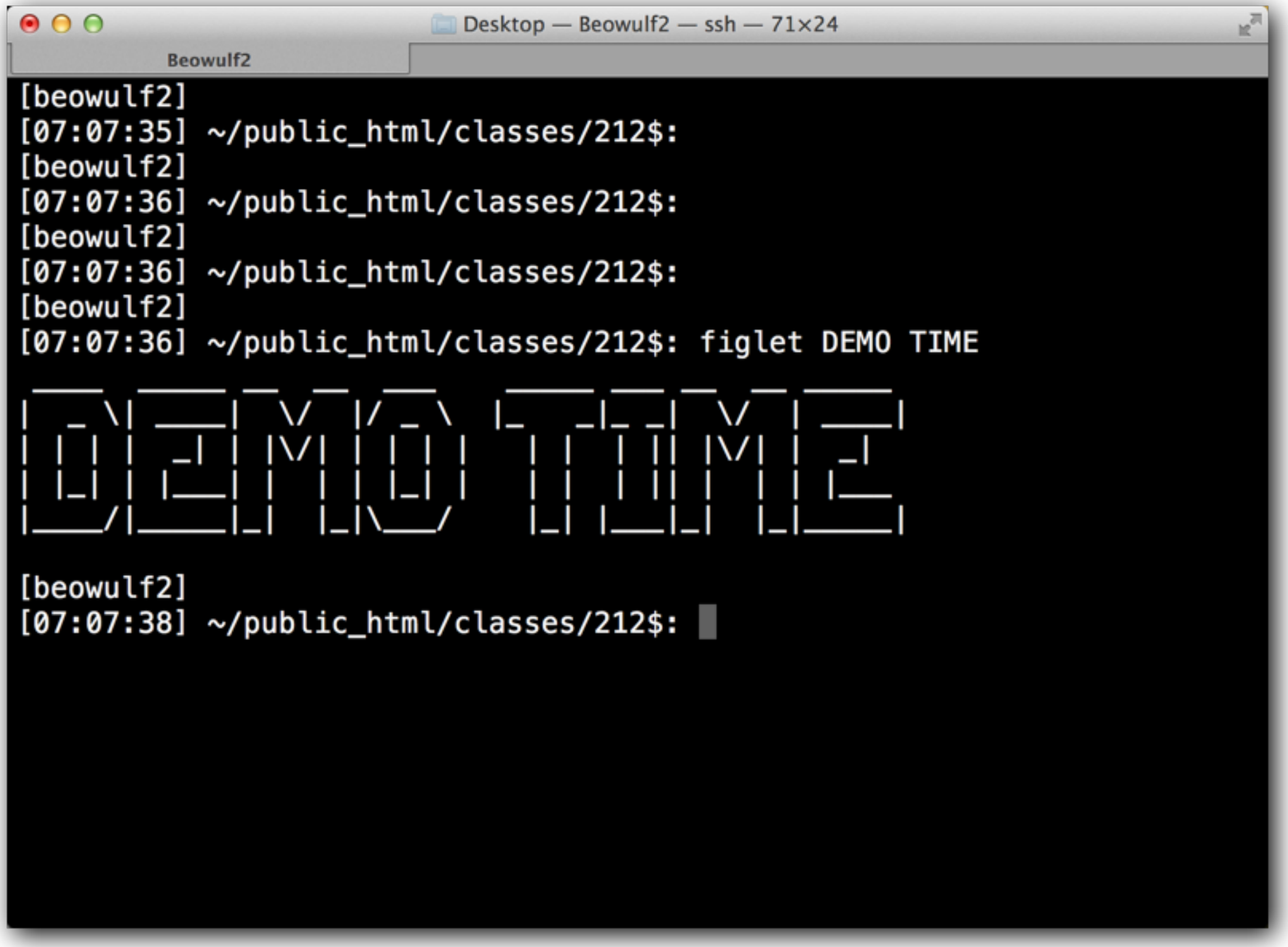
- **So Many leaves!**
- Most of the work in the lower part of the tree, where the leaves are…
- If we could "prune" the tree, we could reduce the amount of work done…

Solution? Cut the Tail-End Recursion!

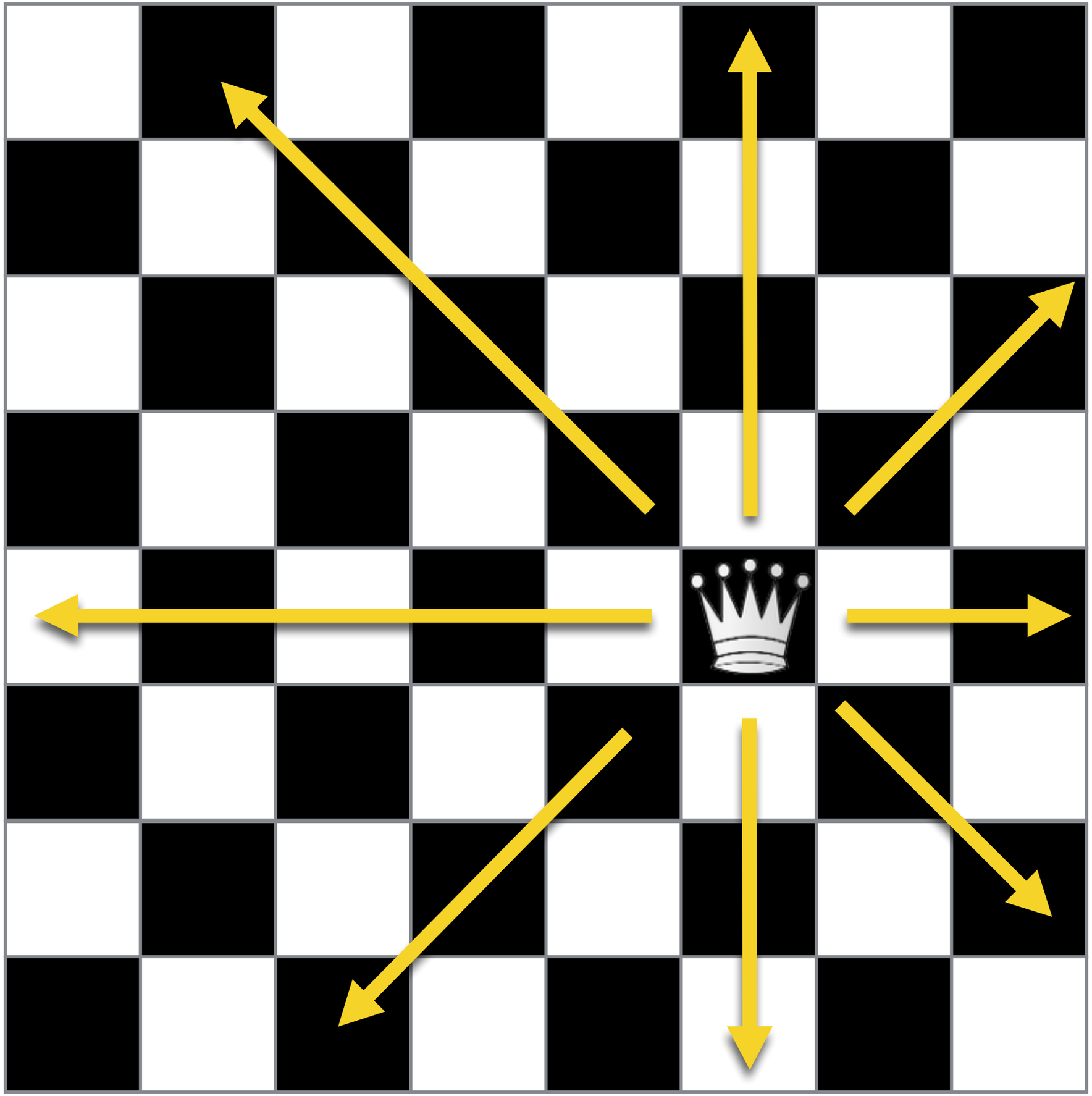
```
private static long computeFibRecursively( int n ) {
    long[] f10 = new long[] {1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89};

    if ( n<= 10 )
        return f10[n];

    return computeFibRecursively( n-1 )
            + computeFibRecursively( n-2 );
}
```


Desktop — Beowulf2 — ssh — 71×24
Beowulf2
[beowulf2]
[07:07:35] ~/public_html/classes/212$:
[beowulf2]
[07:07:36] ~/public_html/classes/212$:
[beowulf2]
[07:07:36] ~/public_html/classes/212$:
[beowulf2]
[07:07:36] ~/public_html/classes/212$: figlet DEMO TIME
[beowulf2]
[07:07:38] ~/public_html/classes/212$:

The N-Queens Problem



Question: Can one put 8 queens on a chess board, such that no two queens can take each other?

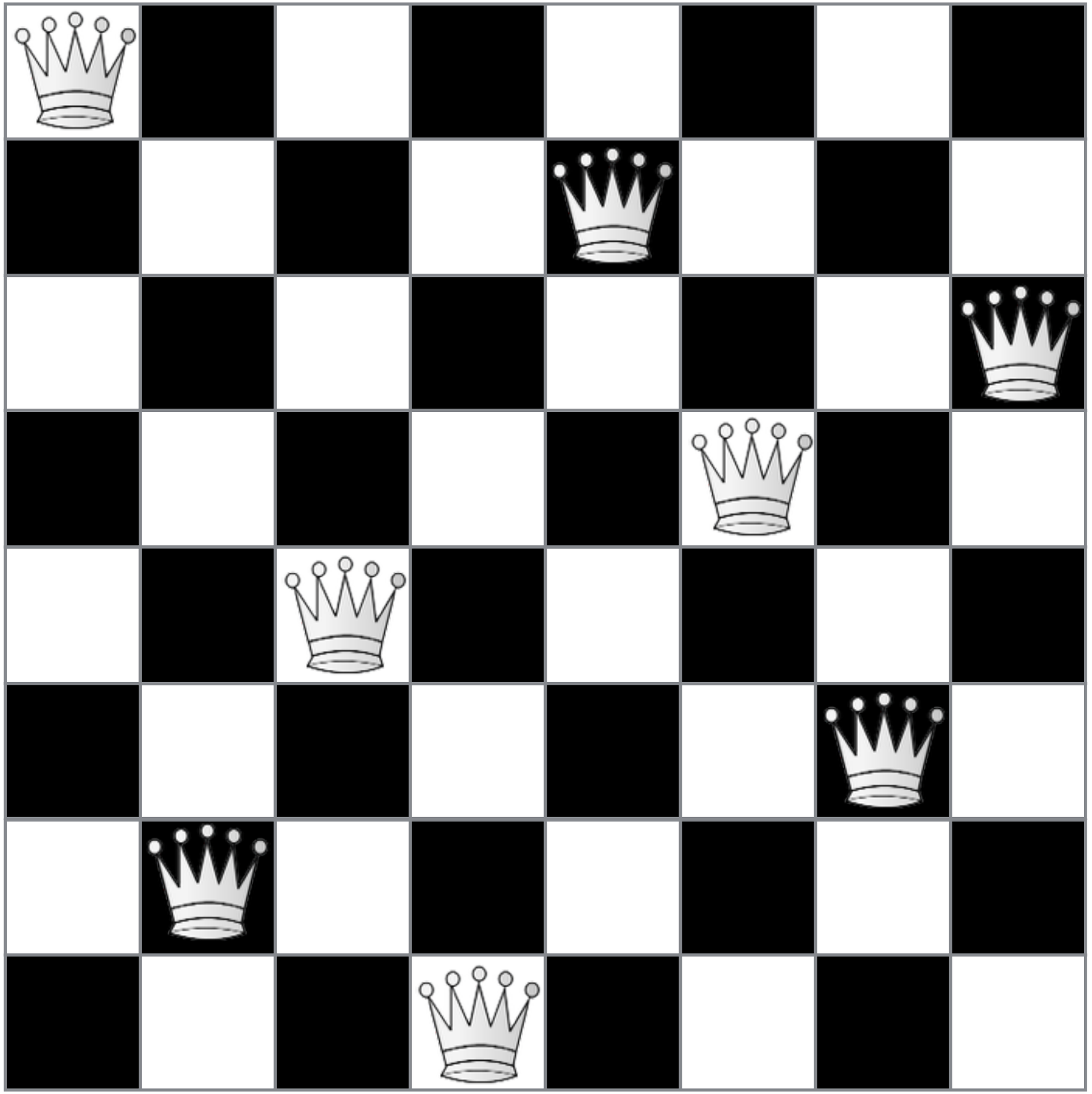
TRY IT!

Questions Before Coding

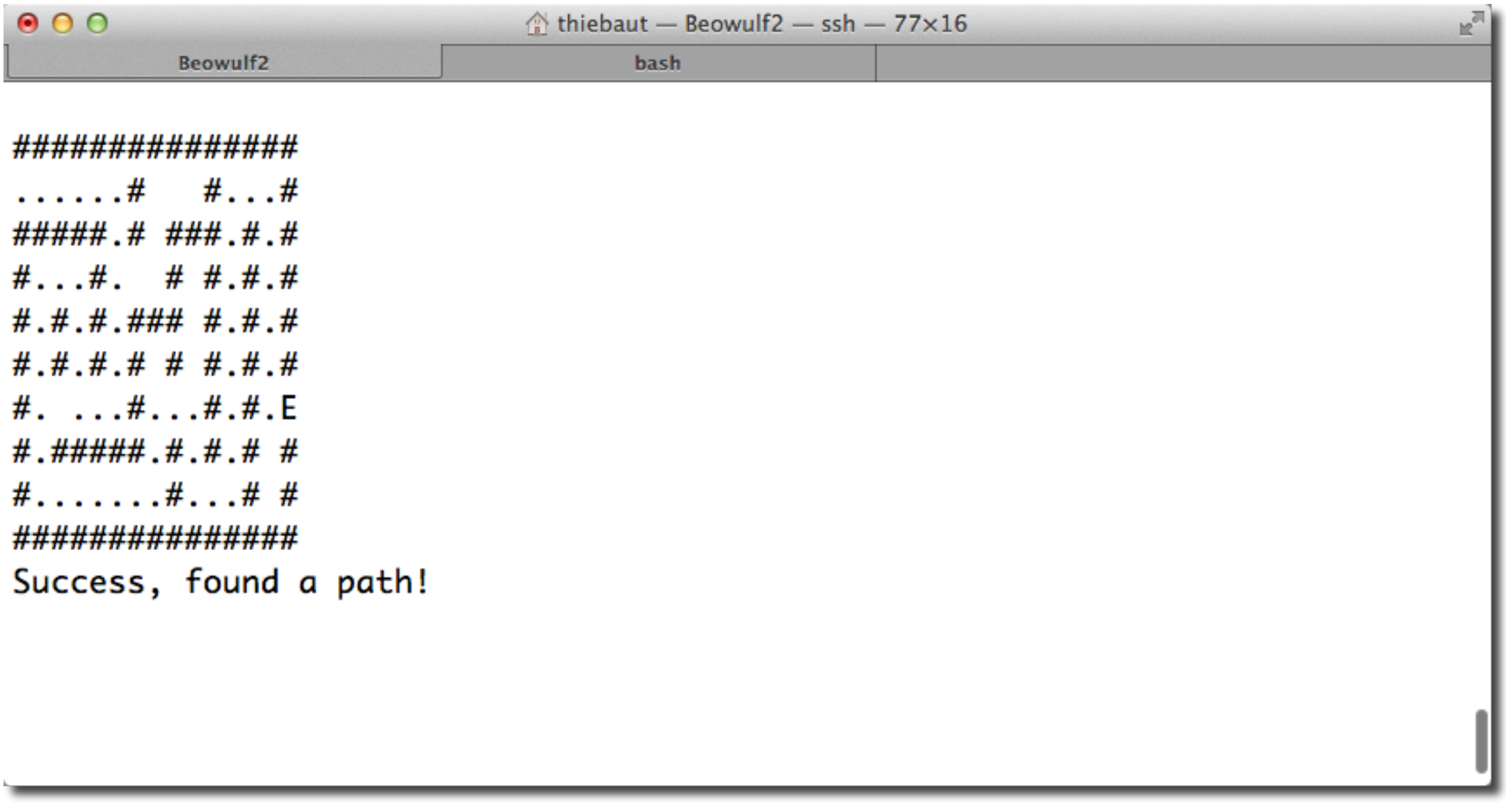
- What data structure can we use?
- How do we represent a placed queen?
- How do we represent a cell "covered" by a queen?
- How do we represent an empty cell?

Important Concept Of the Day

- **Back-Tracking**: the action of returning from recursive exploration of a sub-problem, undoing some computation, selecting a new unexplored path, and starting exploring it recursively.



2D Maze Traversal



Class Exercise

- Think of a recursive way of visiting the maze…
- You have to make sure that we keep exploring until we find the exit
- There might be dead-ends
- There might not be an exit
- We can only see 1 cell of the array at a time…