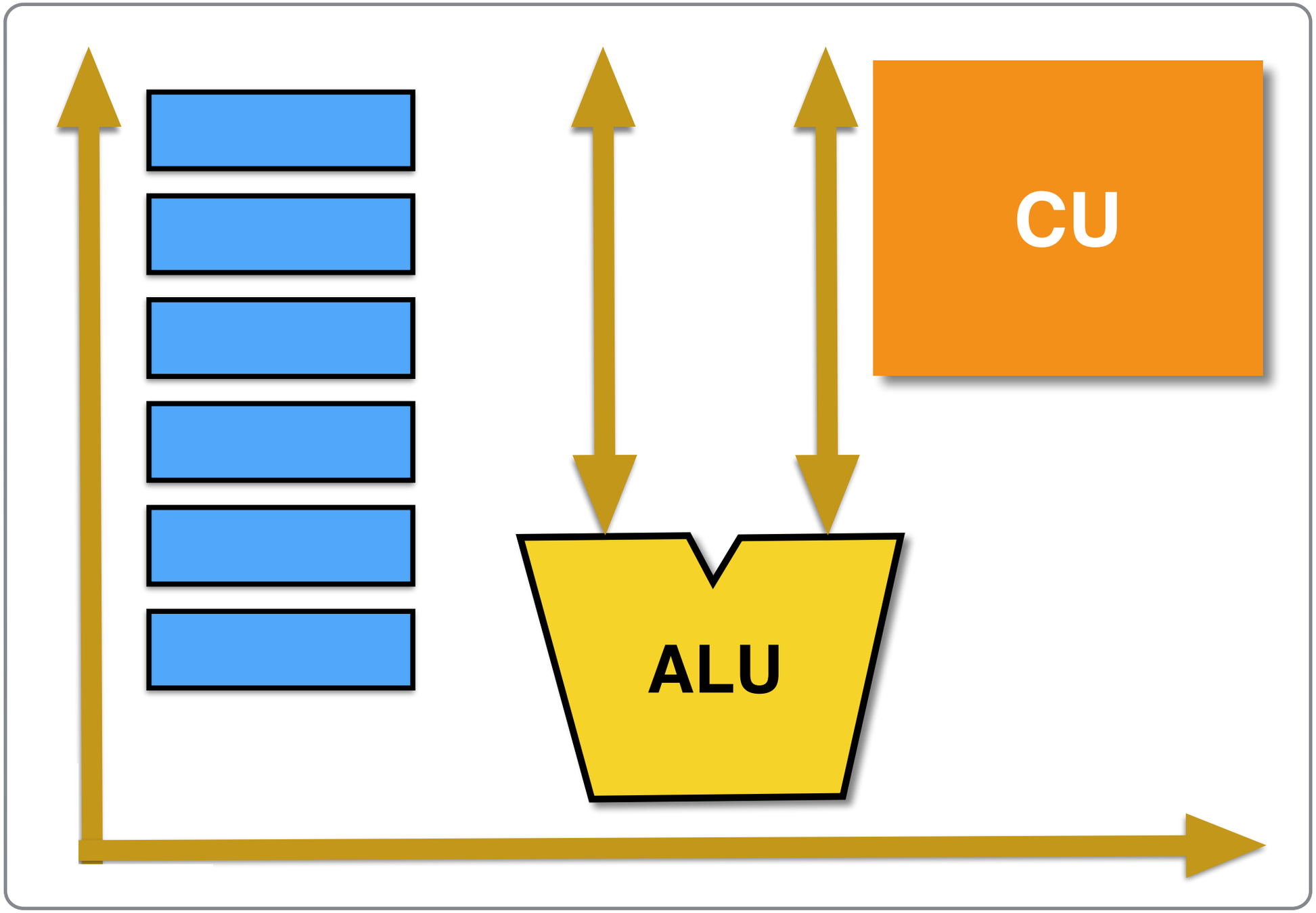
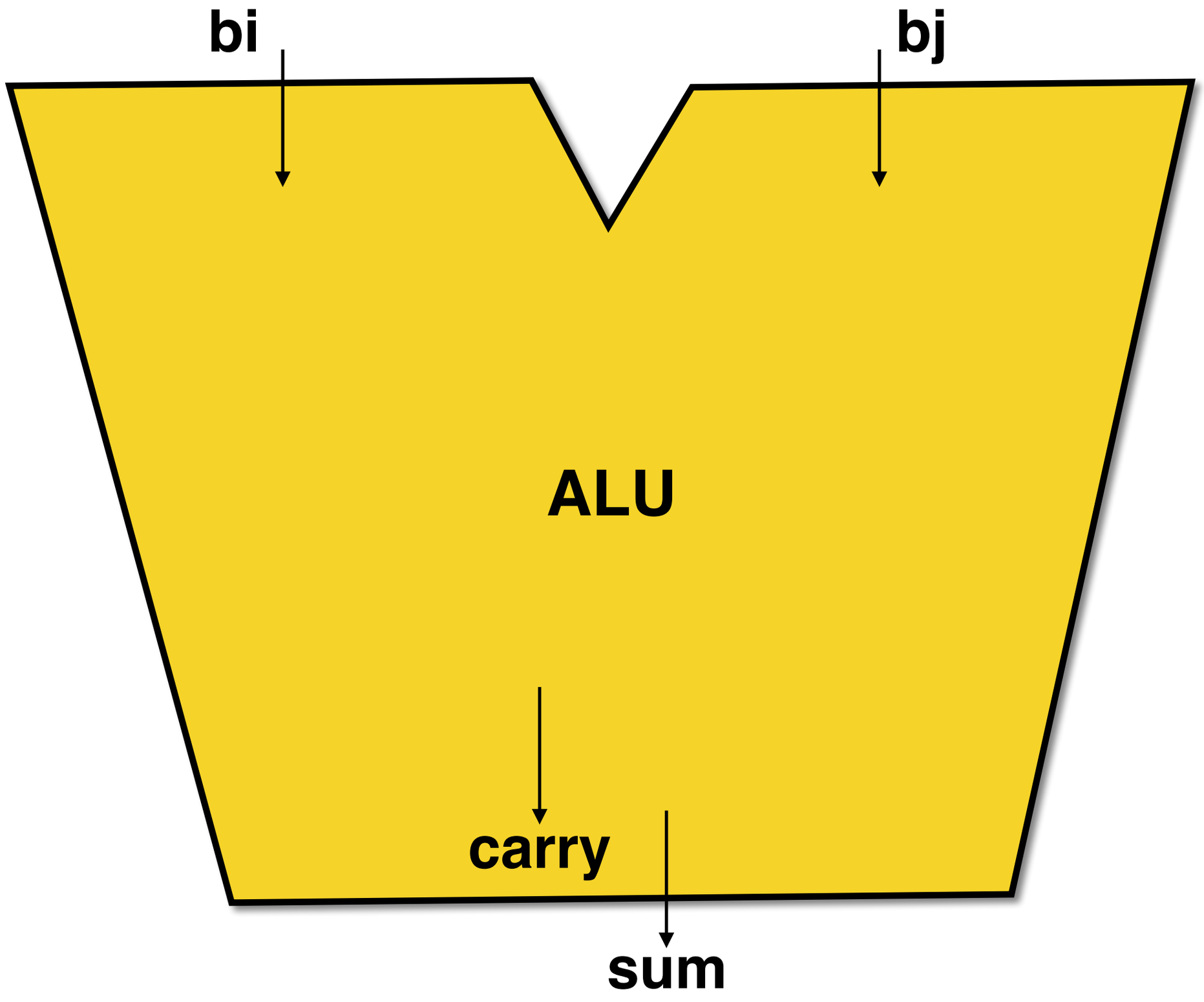
SMITH COLLEGE
COMPUTER SCIENCE

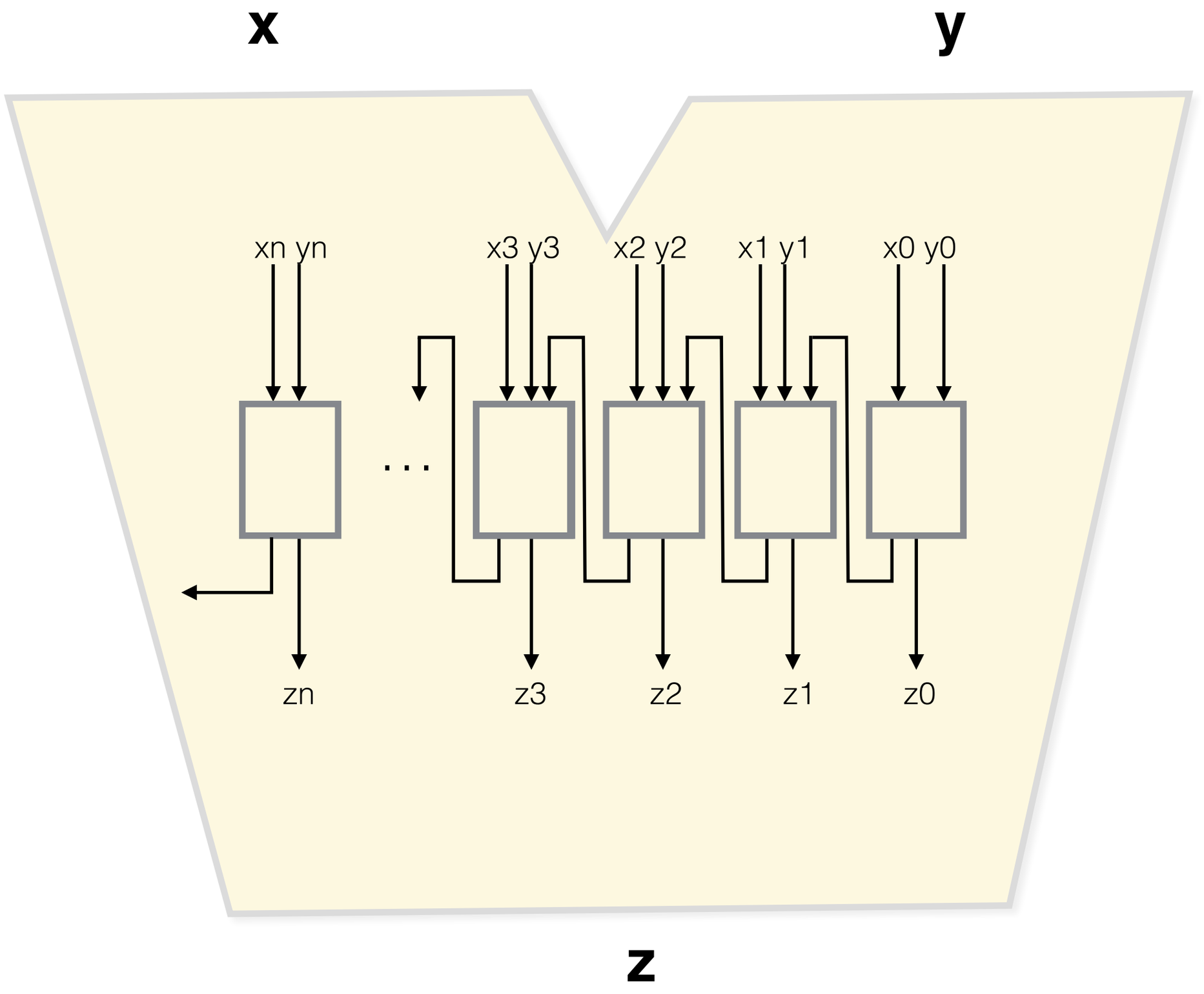
Number Systems

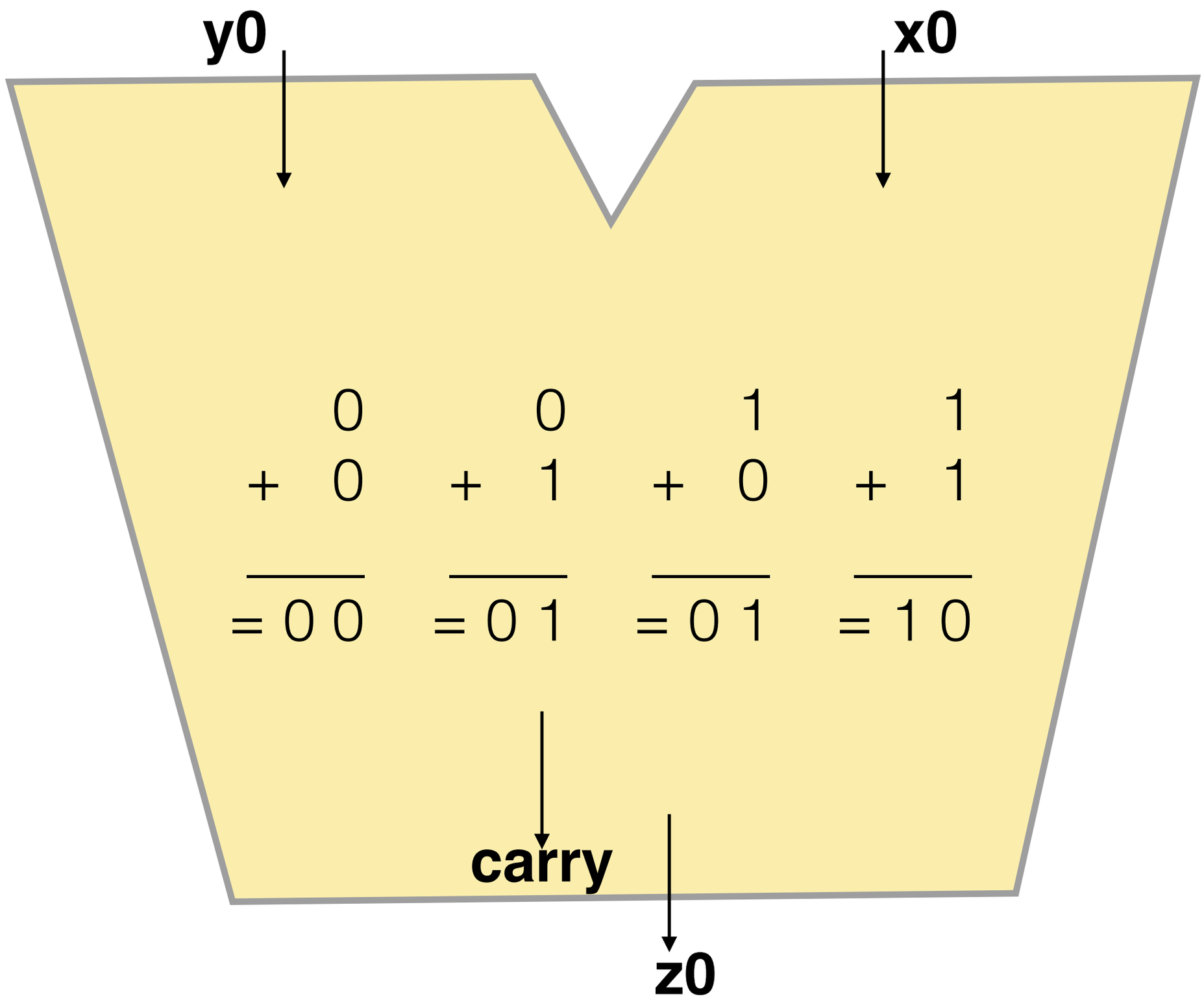
CSC231 — Fall 2014
D. Thiebaut

Processor




bi
bj
ALU
carry
sum


x
y
xn yn
x3 y3
x2 y2
x1 y1
x0 y0
zn
z3
z2
z1
z0
z

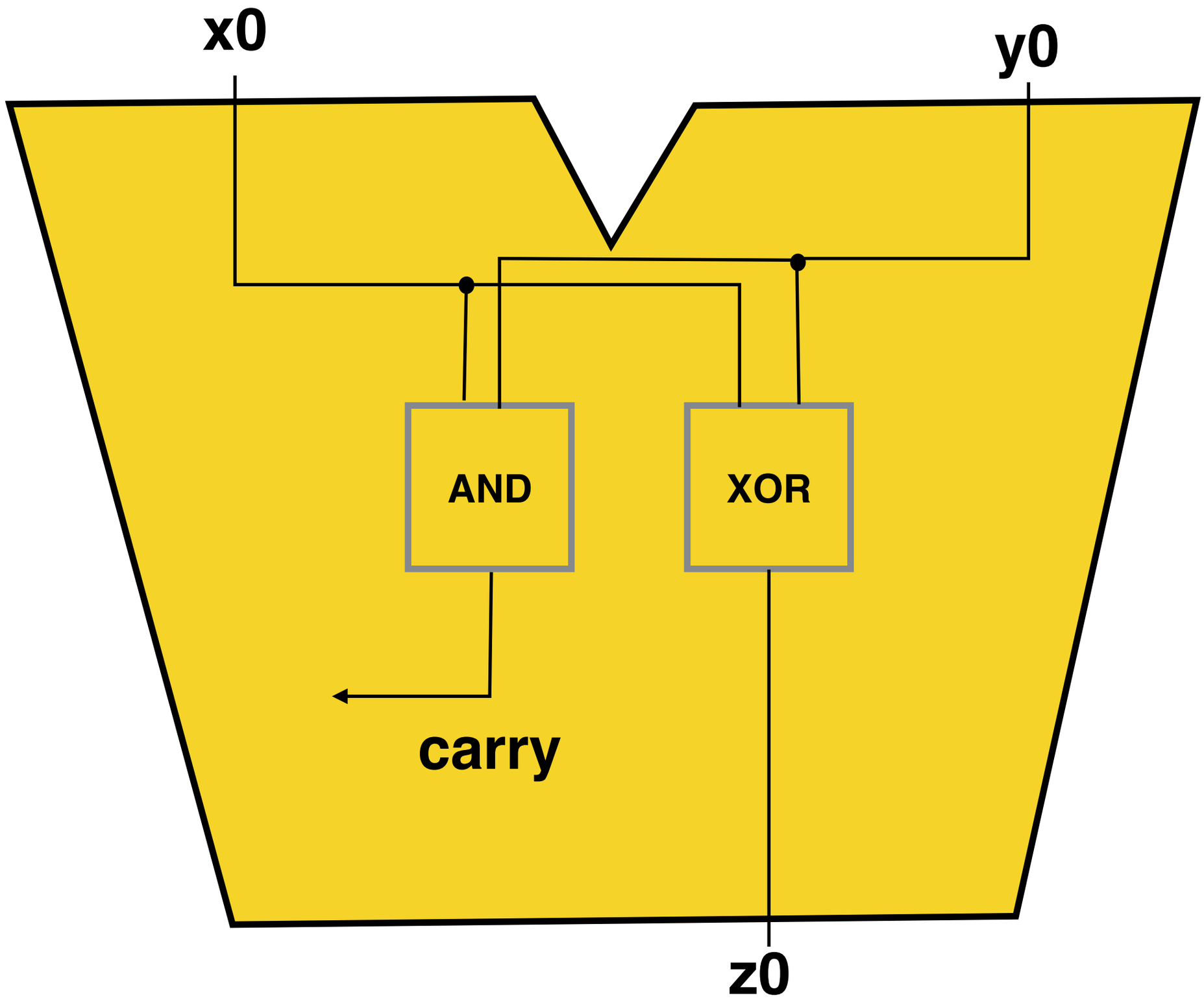

y0
x0
0 0 1 1
+ 0 + 1 + 0 + 1
= 0 0 = 0 1 = 0 1 = 1 0
carry
z0

x0	y0	Carry	y0
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

x0	y0	Carry	z0
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Carry = bi **and** bj

Sum = bi **xor** bj


x0
y0
AND
XOR
carry
z0

Moral of the Story:
Addition is performed by logic operations using *natural* binary numbers…

(*unsigned arithmetic*)

How can we represent signed binary numbers when all we have are **bits** (0/1)?

Whichever system we use should work with the ALU adder…

4-bit Nybble

Binary	Hex	Unsigned Decimal
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	A	10
1011	B	11
1100	C	12
1101	D	13
1110	E	14
1111	F	15

4-bit Nybble

Sign Bit

Binary	Hex	Unsigned Decimal
0 000	0	0
0 001	1	1
0 010	2	2
0 011	3	3
0 100	4	4
0 101	5	5
0 110	6	6
0 111	7	7
1 000	8	8
1 001	9	9
1 010	A	10
1 011	B	11
1 100	C	12
1 101	D	13
1 110	E	14
1 111	F	15

4-bit Nybble

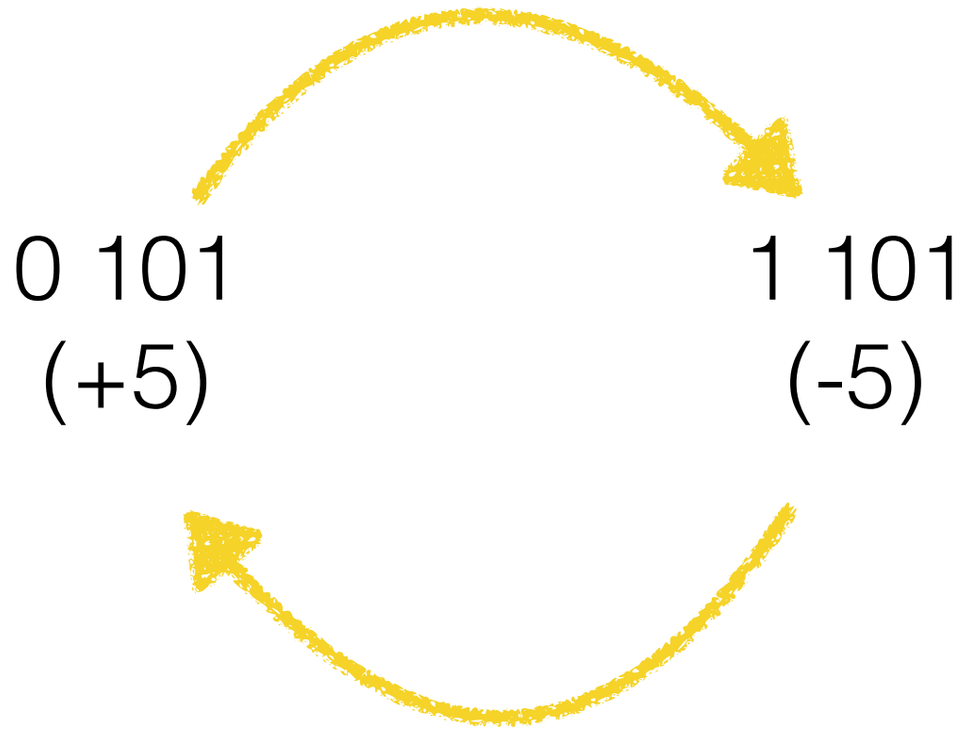
Binary	Hex	Unsigned Decimal
0 000	0	0
0 001	1	1
0 010	2	2
0 011	3	3
0 100	4	4
0 101	5	5
0 110	6	6
0 111	7	7
1 000	8	8
1 001	9	9
1 010	A	10
1 011	B	11
1 100	C	12
1 101	D	13
1 110	E	14
1 111	F	15

Positive Numbers

Negative Numbers

Signed Magnitude Number ***System***

Signed Magnitude Rule: To find the opposite of a positive number, just change its MSB to 1



and vice versa…

4-bit Nybble

Binary	Hex	Unsigned Decimal	**Signed Magnitude**
0 000	0	0	**+0**
0 001	1	1	**+1**
0 010	2	2	**+2**
0 011	3	3	**+3**
0 100	4	4	**+4**
0 101	5	5	**+5**
0 110	6	6	**+6**
0 111	7	7	**+7**
1 000	8	8	**-0**
1 001	9	9	**-1**
1 010	A	10	**-2**
1 011	B	11	**-3**
1 100	C	12	**-4**
1 101	D	13	**-5**
1 110	E	14	**-6**
1 111	F	15	**-7**

Does this System work With the ALU Adder?

Binary	Hex	Unsigned Decimal	**Signed Magnitud**
0 000	0	0	**+0**
0 001	1	1	**+1**
0 010	2	2	**+2**
0 011	3	3	**+3**
0 100	4	4	**+4**
0 101	5	5	**+5**
0 110	6	6	**+6**
0 111	7	7	**+7**
1 000	8	8	**-0**
1 001	9	9	**-1**
1 010	A	10	**-2**
1 011	B	11	**-3**
1 100	C	12	**-4**
1 101	D	13	**-5**
1 110	E	14	**-6**
1 111	F	15	**-7**

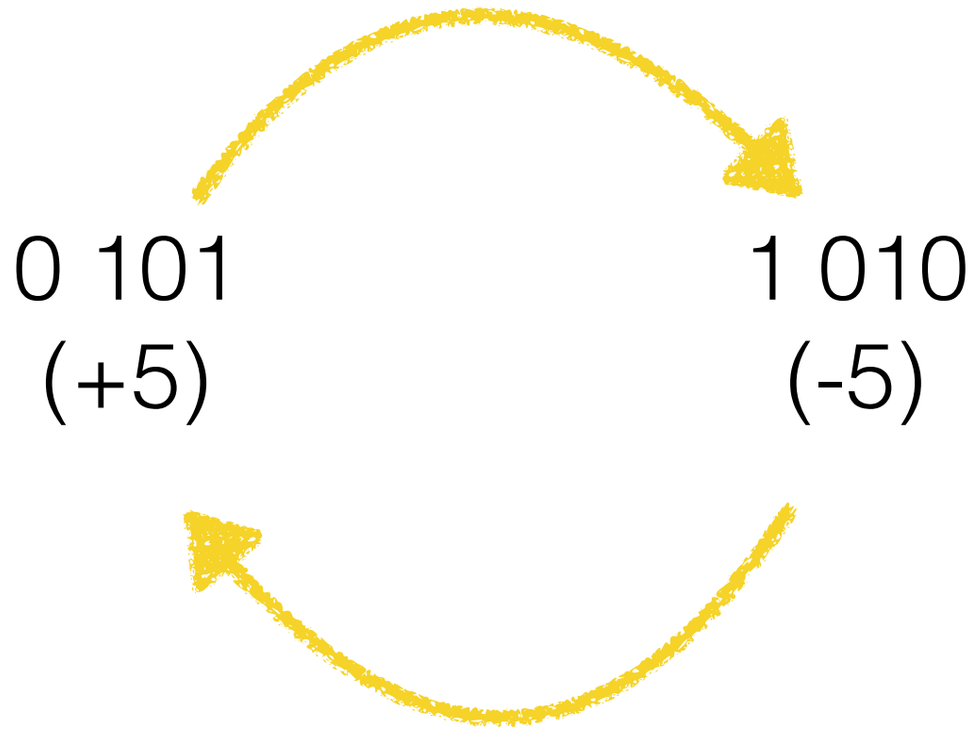
```
   3
+ -3
-----
=  0
```

```
   4
+ -1
-----
=  3
```

1's Complement

Number ***System***

1's Complement Rule: To find the opposite of a positive number, just flip all bits



and vice versa…

4-bit Nybble

Binary	Hex	Unsigned Decimal	**1's Complement**
0 000	0	0	**+0**
0 001	1	1	**+1**
0 010	2	2	**+2**
0 011	3	3	**+3**
0 100	4	4	**+4**
0 101	5	5	**+5**
0 110	6	6	**+6**
0 111	7	7	**+7**
1 000	8	8	**-7**
1 001	9	9	**-6**
1 010	A	10	**-5**
1 011	B	11	**-4**
1 100	C	12	**-3**
1 101	D	13	**-2**
1 110	E	14	**-1**
1 111	F	15	**-0**

Does this System work With the ALU Adder?

Binary	Hex	Unsigned Decimal	**1's Complement**
0 000	0	0	**+0**
0 001	1	1	**+1**
0 010	2	2	**+2**
0 011	3	3	**+3**
0 100	4	4	**+4**
0 101	5	5	**+5**
0 110	6	6	**+6**
0 111	7	7	**+7**
1 000	8	8	**-7**
1 001	9	9	**-6**
1 010	A	10	**-5**
1 011	B	11	**-4**
1 100	C	12	**-3**
1 101	D	13	**-2**
1 110	E	14	**-1**
1 111	F	15	**-0**

```
   3
+ -3
-----
=  0
```

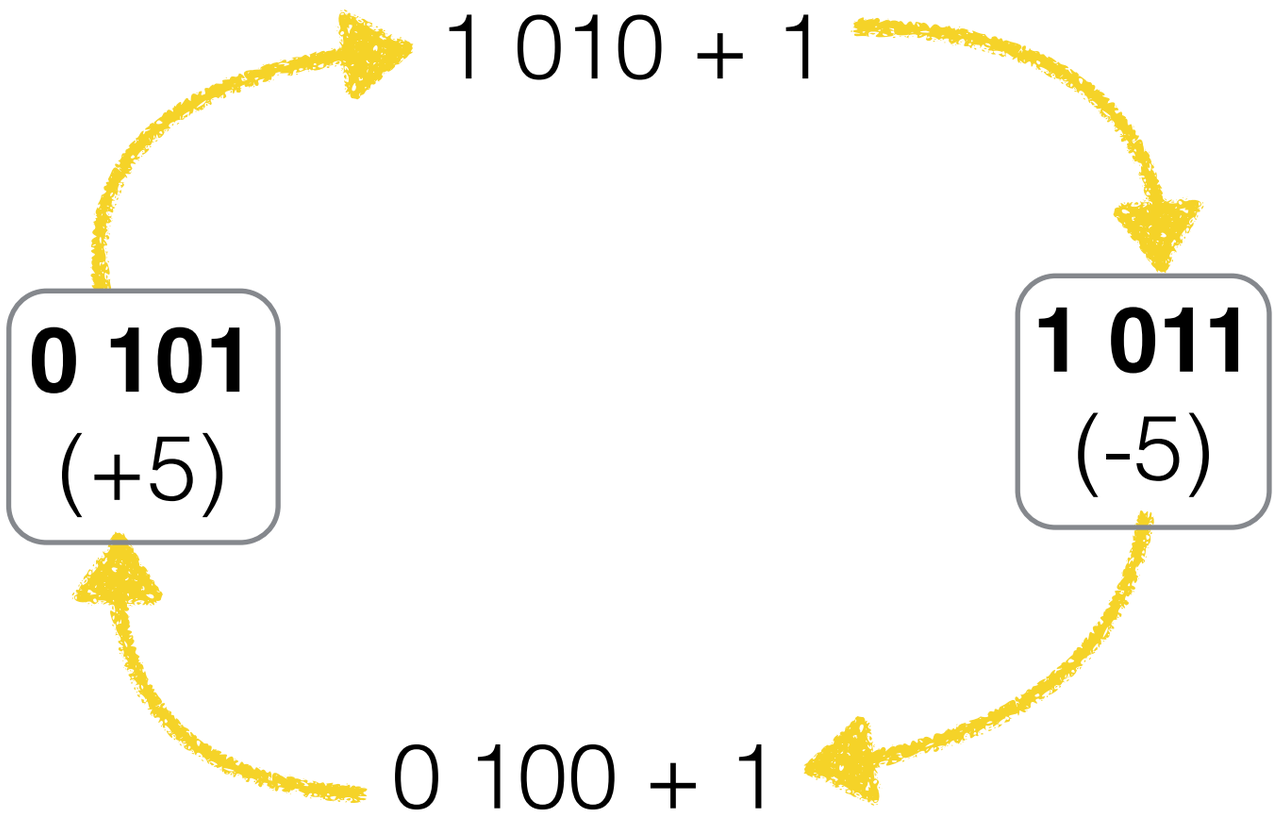
```
   4
+ -1
-----
=  3
```

```
   5
+ -3
-----
=  2
```

2's Complement

Number ***System***

2's Complement Rule: To find the opposite of a positive number, just flip all bits, and add 1



and vice versa…

4-bit Nybble

Binary	Hex	Unsigned Decimal	**2's Complement**
0 000	0	0	**+0**
0 001	1	1	**+1**
0 010	2	2	**+2**
0 011	3	3	**+3**
0 100	4	4	**+4**
0 101	5	5	**+5**
0 110	6	6	**+6**
0 111	7	7	**+7**
1 000	8	8	**-8**
1 001	9	9	**-7**
1 010	A	10	**-6**
1 011	B	11	**-5**
1 100	C	12	**-4**
1 101	D	13	**-3**
1 110	E	14	**-2**
1 111	F	15	**-1**

Does this System work With the ALU Adder?

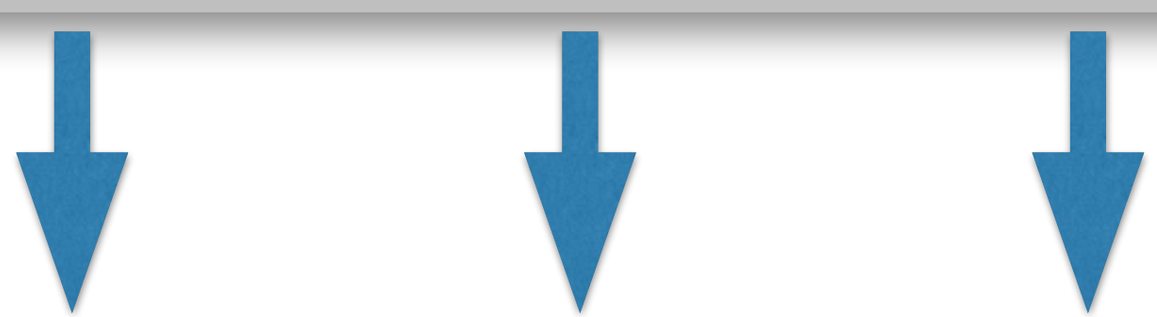
Binary	Hex	Unsigned Decimal	**2's Complement**
0 000	0	0	**+0**
0 001	1	1	**+1**
0 010	2	2	**+2**
0 011	3	3	**+3**
0 100	4	4	**+4**
0 101	5	5	**+5**
0 110	6	6	**+6**
0 111	7	7	**+7**
1 000	8	8	**-8**
1 001	9	9	**-7**
1 010	A	10	**-6**
1 011	B	11	**-5**
1 100	C	12	**-4**
1 101	D	13	**-3**
1 110	E	14	**-3**
1 111	F	15	**-1**

```
   3
+ -3
-----
=  0

   4
+ -1
-----
=  3

   5
+ -3
-----
=  2
```

Interesting Property

- What is the binary representation of **-1** as a byte?
- What is the binary representation of **-1** as a word?
- What is the binary representation of **-1** as a dword?

```
int x = 0x7fffffff - 5;

for ( int i=0; i<10; i++ )
   System.out.println( x++ );
```



Java **ints** are

signed!

Exercises